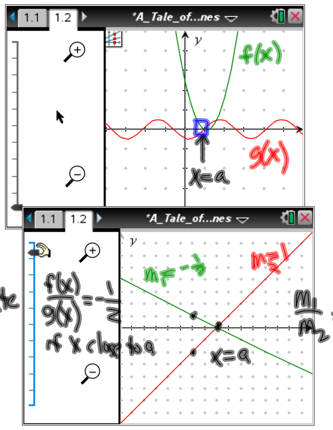
8.2 L'Hopital's Rule

$$\lim_{x\to a}\frac{f(x)}{g(x)} = ?\ \frac{0}{0}$$

$$\boxed{\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}}$$

need $\frac{0}{0}$ indeterminate form



$\frac{m_1}{m_2} = -\frac{1}{2}$

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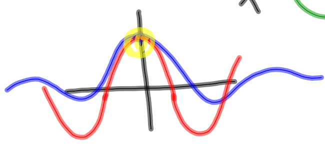
$$\lim_{x\to 0}\frac{\sin(x)}{x} \overset{\frac{0}{0}}{=} \frac{\cos(0)}{1} = \frac{1}{1} = 1$$

graph sin(x)/x and cos(x)/1. How does this support l'hopital's rule?

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\cos x}{1} = 1$$



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$$\lim_{x\to 0}\frac{\sqrt{1+x}}{x} = \infty \quad \left(\frac{\to 1}{\to 0}\right)$$

not $\frac{0}{0}$

$$\lim_{x\to 0}\frac{1}{x} = \pm\infty$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1-x/2}{x^2} \overset{\frac{0}{0}}{=} \lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}}{2x} = \lim_{x\to 0}\frac{-\frac{1}{4}(1+x)^{-\frac{3}{2}}}{2}$$

still $\frac{0}{0}$

$$= -\frac{1}{8}$$

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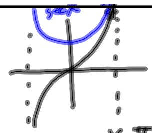
$\frac{\infty}{\infty}$

$$\lim_{x\to\pi/2}\frac{\sec x}{1+\tan x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\frac{\pi}{2}}\frac{\sec x\tan x}{\sec^2 x} = \lim_{x\to\frac{\pi}{2}}\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \lim_{x\to\frac{\pi}{2}}\sin x = 1$$



$$\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}} = \lim_{x\to\infty}\frac{\frac{1}{x}}{2\cdot\frac{1}{2}x^{-\frac{1}{2}}} = \lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty}\frac{\sqrt{x}}{x}$$

$$= \lim_{x\to\infty}\frac{2\frac{1}{\sqrt{x}}}{1} = 0$$

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Indeterminate
$\infty \cdot 0$

$$\lim_{x\to\infty} x \sin\frac{1}{x} = 1$$

$$\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\cos\frac{1}{x}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \cos 0 = 1$$

$\frac{0}{0}$

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$\infty - \infty$ Indeterminate form

$$\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) =$$

$$\lim_{x\to 1} \frac{(x-1)-\ln x}{\ln x\,(x-1)} = \lim_{x\to 1} \frac{\left(1-\frac{1}{x}\right)\cdot x}{\left(\ln x + (x-1)\frac{1}{x}\right)\cdot x}$$

$\frac{0}{0}$

$$\lim_{x\to 1} \frac{x-1}{x\ln x + x - 1}$$

$$\lim_{x\to 1} \frac{1}{x\cdot\frac{1}{x} + \ln x\cdot 1 + 1} = \lim_{x\to 1} \frac{1}{1+\ln x+1} = \frac{1}{2}$$

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1^{∞}

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x = \ln y$$

$\infty \cdot 0$

$$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) = \ln y$$

$$\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \ln y$$

$\frac{0}{0}$

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \ln y$$

$1 = \ln y$

$y = e$

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